

Moreau gradient

Instead of $\nabla f(x)\lambda$, consider computing

$$\nabla \text{env}(\lambda^\top f)(x) = \arg \min_y \lambda^\top f(x+y) + \frac{1}{2} \|y\|_2^2$$

1. for f linear retrieve $\nabla f(x)\lambda$
2. for $\lambda^\top f$ convex \rightarrow grad. of Moreau envelope

Generic template

1. Consider back-propagation computations
2. For each operation, use BP among

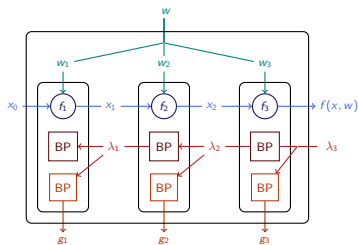
$$\text{GBP} : \nabla f(x)\lambda$$

$$\text{MBP} : \arg \min_y \lambda^\top f(x+y) + \|y\|_2^2/2$$

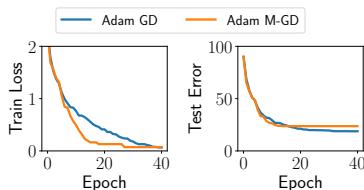
$$\text{IBP} : \arg \min_y \|f(x+y) - f(x) + \lambda\|_2^2 + \gamma \|y\|_2^2$$

MBP: Moreau gradients

IBP: Target propagation,
proximal back-propagation (Frerix et al. 2018)



Generic computational scheme



All-CNN on CIFAR10