On the Convergence of the Iterative Linear Exponential Quadratic Gaussian Algorithm to Stationary Points

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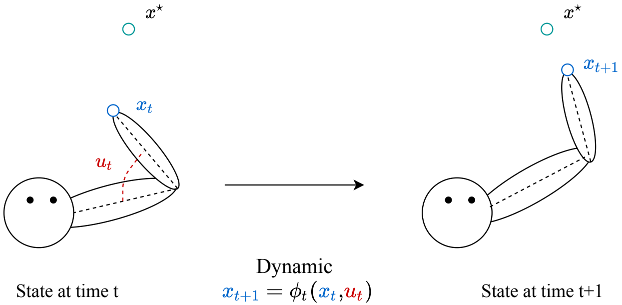
Control Problem



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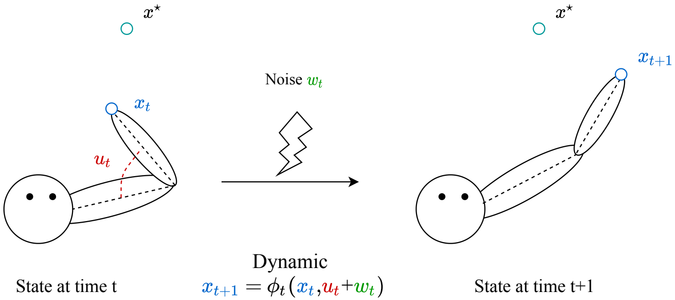

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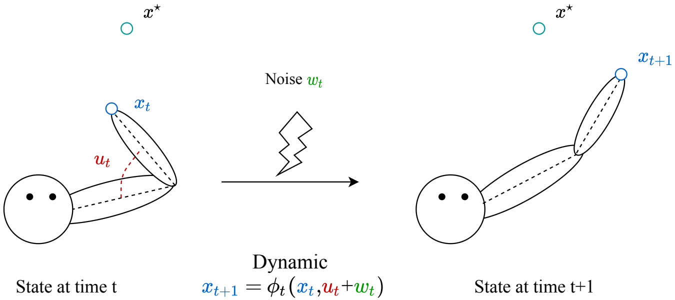

Objective

$$\min_{u} \quad \mathbb{E}_w[h(x(u+w))] + g(u)$$
$$\text{where} \quad x_{t+1}(u+w) = \phi_t(x_t(u+w), u_t + w_t)$$

and $u = (u_0, \ldots, u_{\tau-1})$ are controls, $w = (w_0, \ldots w_{\tau-1})$ are noises.

- State cost $h(x) = \sum_{t=0}^{\tau} h_t(x_t)$,
- Control cost $g(u) = \sum_{t=0}^{\tau-1} g_t(u_t)$,

Risk-Sensitive Objective

Risk Sensitive Objective (Whittle 1981)

$$\min_{u_0,\ldots,u_{T-1}} f_\theta(u) = \left\{ \frac{1}{\theta} \log \mathbb{E}_w[\exp \theta h(x(u+w))] + g(u) \right\}$$

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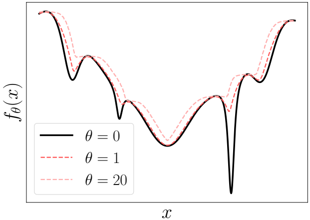
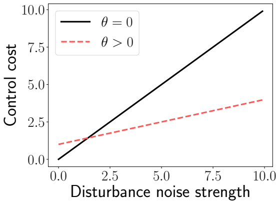


Effect of θ for
$f_\theta(x) = \frac{1}{\theta} \log \mathbb{E}_{w\sim\mathcal{N}(0,1)} \left[\exp \theta F(x+w) \right]$



Expected behavior of
the risk-sensitive controllers.

Iterative Linear Exponential Quadratic Gaussian algorithm

Linear Exponential Quadratic Gaussian (LEQG) **(Whittle 1981)**
For *linear* dynamics, *quadratic* costs, *small enough* θ,
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Questions:

- Does this algorithm converge? Under which assumptions?
- How is the line-search implemented? Is there a principled way?

ILEQG from Optimization Viewpoint

Theoretical Answers **(Roulet et al. 2019)**

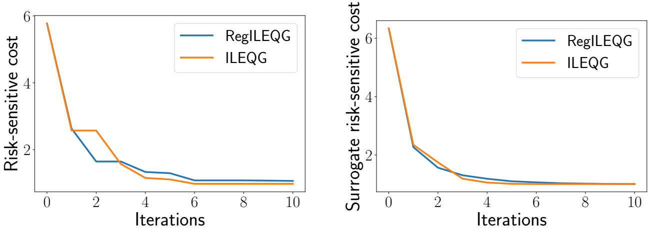
For h, g quadratics, ϕ_t bounded, Lipschitz, smooth

1. Identify surrogate risk-sensitive cost

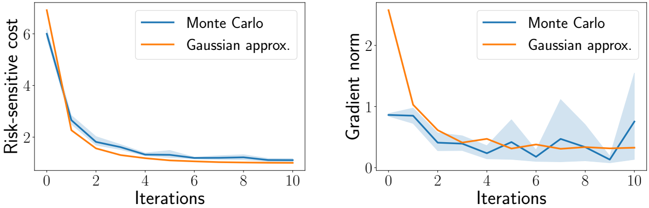
$$\hat{f}_\theta(u) = \underbrace{\frac{1}{\theta} \log \mathbb{E}_w \exp[\theta h(x(u) + \nabla x(u)^\top w)]}_{\hat{\eta}_\theta(u)} + g(u),$$

2. Define regularized ILEQG, named RegILEQG, that minimizes $\hat{f}_\theta(u)$
3. $\hat{\eta}_\theta(u)$ can be computed analytically $\rightarrow$ access to line-search
4. Prove convergence to a near-stationary point of $\hat{f}_\theta$ for small γ_k

Numerical Illustrations



Convergence of ILEQG, RegILEQG



Approximations by surrogate risk-sensitive cost vs Monte-Carlo

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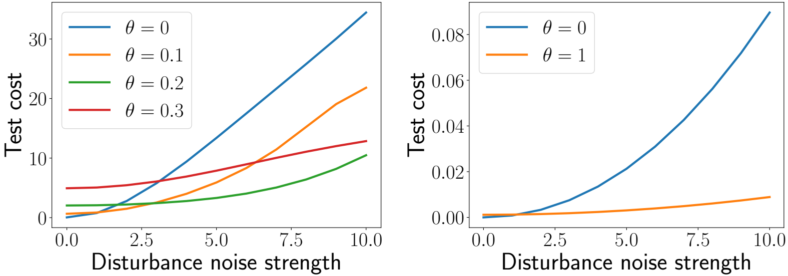

(a) Pendulum. (b) Two-link arm.

Robustness of controllers against disturbance noise.

Code available at https://github.com/vroulet/ilqc

Farshidian, F. & Buchli, J. (2015), 'Risk sensitive, nonlinear optimal control: Iterative linear exponential-quadratic optimal control with Gaussian noise', *arXiv preprint arXiv:1512.07173* .

Roulet, V., Fazel, M., Srinivasa, S. & Harchaoui, Z. (2019), 'On the convergence to stationary points of the iterative linear exponential quadratic gaussian algorithm', *arXiv preprint* .

Whittle, P. (1981), 'Risk-sensitive linear quadratic Gaussian control', *Advances in Applied Probability* **13**(4), 764–777.